

# Formation Games of Reliable Networks

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**Abstract**—We establish a network formation game for the Internet’s Autonomous System (AS) interconnection topology. The game includes different types of players, accounting for the heterogeneity of ASs in the Internet. We incorporate reliability considerations in the player’s utility function, and analyze static properties of the game as well as its dynamic evolution. We provide dynamic analysis of its topological quantities, and explain the prevalence of some “network motifs” in the Internet graph. We assess our predictions with real-world data.

## I. INTRODUCTION

The Internet is a primary example of a large-scale, self organized, complex system. Understanding the processes that shape its topology would provide tools for engineering its future.

The Internet is assembled out of multiple Autonomous Systems (ASs), which are contracted by different economic agreements. These, in turn, compose the routing pathways among the ASs. With some simplifications, we can represent the resulting network as a graph, where two nodes (ASs) are connected by a link if traffic is allowed to traverse through them. The statistical properties of this “Internet graph”, such as the clustering properties, degree distribution etc., have been thoroughly investigated [1]. However, without proper understanding of the mechanism that led to this structure, the statistical analysis alone lacks the ability to either predict the future evolution of the Internet nor to shape its evolution.

A large class of models, a primary example is the “preferential attachment” model [2], use probabilistic rules in order to simulate the network evolution and recover some of its statistical properties. Yet, these models fail to account for many other features of the network [3]. Possibly, one of the main reasons for that is they treat the ASs as passive elements rather than economic, profit-maximizing entities. Therefore, an agent-based approach is a promising alternative.

Game theory is one of the main tools of the trade in estimating the performance of distributed algorithms [4]. It describes the behavior of interacting rational agents and the resulting equilibria. Game theory has been applied extensively to fundamental control tasks in communication networks, such as flow control [5], network security [6], routing [7] and wireless network design [8].

Recently, there has been increased activity in the field of *network formation games*. These studies aim to understand the network structure that results from interactions between rational agents [9], [10]. Different authors emphasized dif-

ferent contexts, such as wireless networks [11] or the inter-AS topology [12], [13]. The theme of such research is to investigate the equilibria’s properties, e.g., establishing their existence and obtaining bounds on the “price of anarchy” and “price of stability”. These metrics measure from above and below, correspondingly, the social cost deterioration at an equilibrium compared with a (socially) optimal solution. Alternatively, agent-based simulations are used in order to obtain statistical characteristics of the resulting topology [14].

Nonetheless, the vast majority of these studies assume that the players are identical, whereas the Internet is a heterogeneous mixture of various entities, such as CDNs, minor ISPs, tier-1 ASs etc. There are only a few studies that have explicitly considered the effects of heterogeneity on the network structure. Some examples include [13], which extends a previous model of formation games for directed networks [9], and in the context of social networks, [15]. The latter describes a network formation game in which the link costs are heterogeneous and the benefit depends only on a player’s nearest neighbors (i.e., no spillovers).

Most of the studies on the application of game theory to networks, with very few exceptions, e.g., [16], focused on static properties of the game. This is particularly true for network formation games. However, it is not clear that the Internet has reached an equilibrium. Indeed, ASs continuously draw new contracts, some merge with others while other quit business. In fact, a dynamic inspection of the inter-AS network suggests that the system may be far from equilibrium. Therefore, a *dynamic* study of an inter-AS network formation game is needed.

In addition, previous work ignores an important requirement that Autonomous Systems has - reliability. Indeed, failures occur, and an AS must face such events. While some game theoretic works addressed reliability in other contexts [17], [18], to the best of our knowledge, there are no works that considered the *topological properties* that emerge in a *heterogeneous, dynamic*, network formation game with *reliability constraints*.

We establish an analytically-tractable model, which explicitly accounts for the heterogeneity of players as well as reliability requirements. We base our model on the heralded Fabrikant model [19], [20], which was recently extended to include heterogeneous players [21]. We model the inter-AS connectivity as a network formation game with *heterogeneous* players that may share costs by monetary transfers. We account

for the inherent bilateral nature of the agreements between players, by noting that the establishment of a link requires the agreement of both nodes at its ends, while removing a link can be done unilaterally. As reliability comes into play, agents may require to be connected to other agents, or to all the other agents in the network, by at least two disjoint paths. We investigate both the static properties of the resulting game as well as its dynamic evolution.

Game theoretic analysis is dominantly employed as a “toy model” for contemplating about real-world phenomena. It is rarely confronted with real-world data. In this study we go a step further from traditional formal analysis, and we do consider real inter-AS topology data analysis to support our theoretical findings.

The main contributions of our study are as follows:

- In the context of network formation games, we provide a theoretical framework that introduces reliability constraints. We discuss both the case of frequent failures, where the fall-back pathways are as frequently used as the main pathways, as well as the case of rare crashes.
- We introduce the concept of “*price of reliability*”, which is defined as the ratio of the social cost with reliability constraints to the social cost with no such additional constraints. Surprisingly, we show that this price can be smaller than one, namely, that the additional reliability requirements may increase the social utility.
- We provide dynamical analysis of topological quantities, and explain the prevalence of some “*network motifs*”, i.e., sub-graphs that appear frequently in the network. Through real-world data, we provide encouraging support to our predictions.

In the next section, we describe our model. We discuss alternative variants that address different failure frequencies or whether utility transfers (e.g., monetary transfers) are allowed or not; we address both the case of allowing utility (i.e., monetary) transfers as well as the case where this is not possible. Next, in Section 3, we provide static analysis. Dynamic analysis is presented in Section 4. In section 5 we compare our theoretical predictions with real-world data on inter-AS topologies. Finally, conclusions are presented in Section 6.

## II. MODEL

We assume that each AS is a player. While there are many types of players, following [21], we aggregate them into two types: *major league* (or *type-A*) players, such as major ISPs, central search engines and the likes, and *minor league* (or *type-B*) players, such as local ISP or small enterprises. Each player, regardless of its type, may form contracts with other players, and should they reach a mutual understanding, a link between them is formed. A player’s strategy is set by specifying which links it is interested in establishing, and, if permissible, the price it will be willing to pay for each. In order to maintain reliable routing pathways, players may be required to sustain at least *two disjoint paths* to other players or a subset of players.

We denote the set of type-A (type B) player by  $T_A$  ( $T_B$ ). A link connecting node  $i$  to node  $j$  is denoted as either  $(i, j)$

or  $ij$ . The total number of players is  $N = |T_A| + |T_B|$ , and we assume  $N \geq 3$ . The *shortest distance* between nodes  $i$  and  $j$  is the minimal number of hops along a path connecting them and is denoted by  $d(i, j)$ . Finally, The degree of node  $i$  is denoted by  $deg(i)$ .

### A. Basic model

Our cost function is based on the cost structure in [19] and [20]. Players are penalized for their distance from other players. First and foremost, players require a good, fast connection to the major players, while they may relax their connection requirements to minor players. Bandwidth usage and delay depends heavily on the hop distance, and connection quality is represented by this metric. Similarly to [21], we weight the relative importance of a major player by a factor  $A > 1$  in the cost function in the corresponding distance term. The link prices represent factors such as the link’s maintenance costs, bandwidth allocation costs etc. Different player types may incur different link costs,  $c_A, c_B$ , due to varying financial resources or infrastructure.

All ASs must maintain access to the Internet in case of a single link failure. This is tantamount to the requirement that all the players must have at least two *disjoint* paths to each other node. Nevertheless, if either link prices are high, crash frequencies are low or the content of a minor AS is of little value, players may relax their reliability requirements and demand the establishment of disjoint paths only to the major players. This is represented in the cost function by a control parameter  $\tau$ , which is set to one if two disjoint paths are required to all nodes, and zero if the requirement holds for (other) nodes of major players only. Conversely, if failures are often, then the regular and backup paths (in the corresponding pair of disjoint paths) are used almost as frequently. As such, they must be weighted the same in the cost function. Therefore, the distance cost is composed of two terms, one represents the distance along the primary path and the other represents the distance along the backup path. The relative weight of these two terms is set by a parameter  $\delta$ . If failures are frequent and the likelihood of using either route is the same, we have  $\delta = 1$ . However, if failures are rare, traffic will be mostly carried across the shorter path. Therefore its length should carry more weight in the cost than the length of the backup route, hence  $\delta \ll 1$ . This motivates the following cost function.

**Definition 1.** Two paths,  $R_{(i,j)} = (i, x_1, x_2, \dots, j)$  and  $R'_{(i,j)} = (i, x'_1, x'_2, \dots, j)$  are disjoint if they have no node in common, namely if the unordered sets satisfy

$$\{x_1, x_2, \dots\} \cap \{x'_1, x'_2, \dots\} = \emptyset.$$

The cost function  $C(i)$  of node  $i$  of type  $\beta \in \{A, B\}$ , is defined as:

$$C_\beta(i) \triangleq deg(i) \cdot c_\beta + \frac{A}{1+\delta} \sum_{j \in T_A} (d(i, j) + \delta d'(i, j)) + \tau \cdot \frac{1}{1+\delta} \sum_{j \in T_B} (d(i, j) + \delta d'(i, j))$$

$$+ (1 - \tau) \sum_{j \in T_B} d(i, j)$$

where  $d(i, j)$  and  $d'(i, j)$  are the lengths of a pair of disjoint paths between  $i, j$  that minimizes the cost function.  $d(i, j)$  denotes the length of the shorter path. Formally, denote a pair of disjoint paths connecting player  $i$  and player  $j$  as  $(R_{(i,j)}, R'_{(i,j)})_\alpha$ , where  $d_\alpha(i, j)$  ( $d'_\alpha(i, j)$ ) is the length of shorter (correspondingly, longer) path. Set

$$(\hat{R}_{(i,j)}, \hat{R}'_{(i,j)}) = \arg \min_{(R_{(i,j)}, R'_{(i,j)})_\alpha} C_\beta(i)$$

then  $d(i, j) = \|\hat{R}_{(i,j)}\|$  and  $d'(i, j) = \|\hat{R}'_{(i,j)}\|$ .

If there is not pair of disjoint path player  $i$  and player  $j$ , then  $d'(i, j) = \mathcal{Q}$ , with  $\mathcal{Q} \rightarrow \infty$ . If there is not a path connecting players  $i$  and  $j$  we also have  $d(i, j) = \mathcal{Q}$ .

For convenience, we set  $c \triangleq (c_A + c_B)/2$ . We assume  $c_A \leq c_B$ . The social cost is the sum of individual costs,  $\mathcal{S} = \sum C_\beta(i)$ . We denote the optimal (minimal) social cost as  $\mathcal{S}_{\text{optimal}}$ , and the social cost at the optimal stable solution is  $\tilde{\mathcal{S}}_{\text{optimal}}$ . The *price of stability* is the ratio between the social cost at the best stable solution and its value at the optimal solution, namely  $PoS = \tilde{\mathcal{S}}_{\text{optimal}}/\mathcal{S}_{\text{optimal}}$ . Similarly, denote by  $\tilde{\mathcal{S}}_{\text{pessimial}}$  the highest social cost in an equilibrium. Then, the *price of anarchy* is the ratio between the social cost at the worst stable solution and its value at the optimal solution, namely  $PoA = \tilde{\mathcal{S}}_{\text{pessimial}}/\mathcal{S}_{\text{optimal}}$ .

Note that the requirement of disjoint *node* paths generalizes the requirement of disjoint *link* paths and protects against link failures within an Autonomous System. All of our results apply to both notions of disjoint path (except a refinement of Theorem 15; see the discussion there). For simplicity and generality we shall use the notion of a disjoint node paths.

**Definition 2.** We denote the change in cost of player  $i$  as after the addition (removal) of a link  $(j, k)$  by  $\Delta C(i, E + jk) \triangleq C(i, E \cup (j, k)) - C(i, E)$  (correspondingly,  $\Delta C(i, E - jk) \triangleq C(i, E) - C(i, E \setminus (j, k))$ ). The abbreviation  $\Delta C(i, jk)$  is often used.

If  $\delta = 1$ , then the two routing pathways are used the same. In this case, the shortest cycle length  $d(i, j) + d'(i, j)$  is the relevant quantity that appears in the cost function. This can be found in polynomial time by using Suurballe's algorithm [22], [23]. However, if  $\delta \ll 1$ , routing will occurs along two disjoint paths, such that the length of the shortest between the two is shortest (among all pairs of disjoint paths). Although the complexity of finding this pair is NP-Hard, first finding the shortest path and then finding the next shortest path is a heuristic that works remarkably well, both in the real-world data analysis and on the networks obtained in the theoretical discussion. The reason behind this is that, when failures are rare, information is predominantly routed along the shortest path. When players are required to establish a fall-back route, they will establish a path that is disjoint from the *current* routing path, namely the shortest one.

The establishment of a link requires the bilateral agreement of the two parties at its ends, while removing a link can be done unilaterally. This is known as a *pairwise-stable* equilibrium [10], [16].

**Definition 3.** The players' strategies are *pairwise-stable* if for all  $i, j \in T_A \cup T_B$ , the following hold:

- a) if  $ij \in E$ , then  $\Delta C(i, E - ij) > 0$ ;
- b) if  $ij \notin E$ , then either  $\Delta C(i, E + ij) > 0$  or  $\Delta C(j, E + ij) > 0$ .

The resulting graph is referred to as a *stabilizable* graph.

The additional reliability requirements result in additional link expenses, as for example, the degree of every node needs to be at least two. The *price of reliability* is the ratio between the optimal social cost under the additional survivability constraint to the optimal social cost when the additional constraints are removed.

**Definition 4.** The cost function,  $C_\beta^{(\text{bare})}(i)$ , of node  $i$  of type  $\beta \in \{A, B\}$ , is obtained by setting  $\delta = 0, \tau = 0$  in Definition 1 and requiring the existence of only a *single* path from player  $i$  to any other players in the network. Denote the optimal social cost without the additional survivability requirement in a pairwise stable equilibrium as  $\tilde{\mathcal{S}}_{\text{optimal}}^{(\text{bare})}$ . The *price of reliability* (*PoR*) is the ratio between the optimal value of the social costs among the set of stable equilibria,  $PoR = \tilde{\mathcal{S}}_{\text{optimal}}/\tilde{\mathcal{S}}_{\text{optimal}}^{(\text{bare})}$ .

Surprisingly, we shall show that there exist scenarios in which reliability requirements *increase* the social utility, so that the price of reliability can be smaller than one.

## B. Utility transfer

Thus far, it was implicitly assumed that utility transfer is not feasible. Nevertheless, often players are able to transfer utility, for example via monetary transactions. An extended model that incorporates such transfers is introduced by allowing for a monetary transaction in which player  $i$  pays player  $j$  some amount  $P_{ij}$  iff the link  $(i, j)$  is established [21]. Player  $j$  sets some minimal price  $w_{ij}$  and should  $P_{ij} \geq w_{ij}$  the link is formed.

**Definition 5.** The cost function of player  $i$  when monetary transfers are allowed is  $\tilde{C}(i) \triangleq C(i) + \sum_{j, ij \in E} (P_{ij} - P_{ji})$ .

We recall the observation in [21] that, without transfers, a link will be established only if *both* parties,  $i$  and  $j$ , reduce their costs,  $C(i, E + ij) < 0$  and  $C(j, E + ij) < 0$ . But, when monetary transfers are allowed, an edge will be established if (and only if) the relaxed condition  $\Delta C(i, E + ij) + \Delta C(j, E + ij) < 0$  holds. In game theoretic terms, this condition is equivalent to the requirement that the *core* of the two players game is non-empty.

**Corollary 6.** When monetary transfers are allowed, the link  $(i, j)$  is established iff  $\Delta C(i, E + ij) + \Delta C(j, E + ij) < 0$ . The link is removed iff  $\Delta C(i, E - ij) + \Delta C(j, E - ij) > 0$ .

In the remainder of the paper, whenever monetary transfers are feasible, we will state it explicitly, otherwise the basic model (without transfers) is assumed.

### III. STATIC ANALYSIS

We shall now analyze the properties of stable equilibria, such as the *price of anarchy*, which is the ratio between the social cost at the worst stable solution and its value at the optimal solution, and the *price of stability*, which is the ratio between the social cost at the best stable solution and its value at the optimal solution. We shall further discuss topological properties that emerge from our analysis.

It was shown in [19] that if  $c < 1$  the only stable solution is a clique and in [21] it was shown that if  $c_A < A$  then the major players form a clique. One may have guessed that reliability requirements, which generally induce the creation of additional, backup edges, would ease the formation of the clique. The next proposition shows that this naive assumption is wrong, and in fact, as the frequency of failure increases, it becomes increasingly difficult to maintain the major player's clique. Consider a dense set, in which every player may access all the other players within two hops by a at least two disjoint paths. A direct link between two players only reduces their mutual distance by one, and does not affect any other distance. If this link fails often, it may be used only partially, and it may not be worthy to pay its cost. Hence, in this setting, counter intuitively, frequent failures end up with a sparser network.

**Proposition 7.** *Assume the frequency of failures is high, namely  $\delta = 1$ . Then, the type-A players form a clique if and only if  $c_A < A/2$ . Allowing monetary transfers does not change the result.*

*Proof:* We consider a major player's clique and ask under which conditions the removal of a link is a worthy move. Consider an edge  $(i, j)$  in this clique. Since only the shortest distance between players  $i$  and  $j$  is affected, and is increased by one, the type-A players clique is stable if and only if  $A/(1 + \delta) < c_A$ . ■

As the major players (tier-1 AS) form a densely connected set, a clique-like subgraph, in the rest of the paper we shall only consider the case where  $c_A < A/2$ . We also assume, trivially, that  $c_B > 1$ , as otherwise the only stabilizable network is a clique.

The next proposition describes a scenario in which, surprisingly, the additional reliability constraints *reduce* the social cost.

**Proposition 8.** *Assume  $1 < c_A < A/2$  and symmetric reliability requirements, namely  $\tau = 1$ . Then, the optimal network is composed of a type-A clique, where all the type B nodes are connected to all members of the type-A clique, as depicted in Fig. 1. This network is not stabilizable, and  $PoS > 1$ . Nevertheless, for  $|T_B| \gg |T_A| \gg 1$ , we have  $PoS \rightarrow 1$ . In addition, the Price of Reliability is smaller than one.*

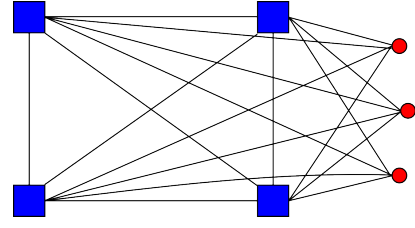


Figure 1. The optimal solution when  $1 < c_A < A/2$  and  $\tau = 1$ , according to Prop. 8. The type-B players are in red circles. The type-A clique is in blue squares.

The main idea behind this result is that in the optimal, yet unstable solution, every minor player establishes a link with all the major players (Fig. 1). This configuration is unstable as it is over-saturated with links, and the optimal stable solution is obtained by diluting this network so that every minor player will connect to just two major players. If the reliability requirements are further removed, then additional dilution occurs, increasing the social cost. In other words, the stable configuration is under-saturated with edges, and the additional survivability requirements facilitate the formation of additional links.

*Proof:*  $\bar{c}_A \leq \bar{c}_A$  We shall now prove the first part of this theorem, namely that the network described in Fig. 1, is optimal in terms of social cost. In this network, every type-B player is connected to every type-A player. We denote this network by  $G$ .

First, consider a network in which the type-A players are not in a clique. Then, there exists two players  $i, j \in T_A$  such that  $d(i, j) \geq 2$ . By establishing a link  $(i, j)$  the change in social cost is

$$\Delta S \leq -2 \left( \frac{A}{1 + \delta} - c_A \right) < 0.$$

Therefore a network which includes this link has a lower social cost. Hence, in the optimal network, all the type-A players are in clique. A similar calculation for a missing link between player  $i \in T_A$  and  $j \in T_B$  shows that

$$\Delta S \leq - \left( \frac{A + 1}{1 + \delta} - c_A - c_B \right) < 0$$

and establishing this link reduces the social cost as well. In conclusion, in an optimal network a type-A player is connected to all the other players.

Consider node  $i \in T_B$ . Its distance from every other node  $j \in T_B$  is  $d(i, j) = 2$  and  $d'(i, j) = 2$ . The minimal distances between two minor players  $i, j \in T_B$  are  $d(i, j) = 1$  and  $d'(i, j) = 2$  whereas in  $G$  we have  $d(i, j) = d'(i, j) = 2$ . Assume that a network with a lower social cost exists, and denote it by  $G'$ . In  $G'$  there must exist a link between two type-B players,  $i, j \in T_B$ . However, removing this link reduces the social cost, as

$$S(E + ij) - S(E) = 2 \left( \frac{A}{1 + \delta} - c \right) > 0$$

Therefore,  $G'$  is not optimal, in contradiction to our assumption. This proves that  $G$  is optimal.

Next, we are going to show  $G$  is unstable. Consider  $i \in T_B$  and  $x \in T_A$ . By removing  $(i, x)$  we cost change of player  $x$  is

$$\Delta C(x, E - ix) = -2 \left( \frac{1}{1+\delta} - c \right) > 0$$

And therefore it is beneficial for player  $x$  to remove this link. Hence, this network is not stabilizable.

Finally, we are going to show that the  $PoR < 1$ . The outline is as follows. First, we are going to show that the network depicted in Fig. 3 is stable. The optimal stable network (without reliability requirements) in this parameter regime was explicitly derived in [21]. We are going to show that the cost in the latter network is higher than the former, which bound the  $PoR$  from above by a value smaller than one.

Consider a network in which all the minor players are connected to two major players,  $x, y \in T_A$ . We are going to show that this network is stable. Clearly, as shown before, the type-A clique is stable. In addition, for any player  $i \in T_B$  neither  $x, y$  nor  $i$  has the incentive to remove either  $(x, i)$  or  $(y, i)$  as it would violate the reliability requirement and would lead to unbounded cost. Hence, this network is stable. Denote this network by  $\tilde{G}$ . In [21] it was shown that the optimal stable network (without reliability requirements) in this configuration is the network in which all type-A players form a clique, and all the type-B players are connected to a single type-A player. We denote the latter network by  $G''$ . We have

$$S(\tilde{G}) - S(G'') = |T_B| \left( \frac{A+1}{1+\delta} - c_A - c_B \right) < 0.$$

But,

$$PoR = \frac{\tilde{S}_{optimal}}{\tilde{S}_{optimal}^{(bare)}} \leq \frac{S(\tilde{G})}{S(G'')} = 1 + \frac{S(\tilde{G}) - S(G'')}{S(G'')} < 1.$$

This concludes the proof.  $\blacksquare$

In conclusion, if the failure frequency is high, the survivability requirements will induce dilution of the clique of major players. However, the opposite effect occurs along the graph cut-set between the set of minor players and the set of major players, where the additional constraints lead to an increased number of links connecting major and minor players.

So far we have assumed that the reliability requirements are symmetric. As explained in the Introduction, in some cases it is reasonable to assume that players will require a backup route only to the major players, i.e., non-symmetric reliability constraints. We shall now show that in this case, the social cost may deteriorate considerably. Hence, from a system designer point of view, it is much more important to incentivize a configuration where the reliability requirements are symmetric, than to reduce failure frequency globally. This result stems from the following lemma.

**Lemma 9.** *If  $\tau \neq 0$  then for every two nodes  $i, j$ ,  $d(i, j) \leq d'(i, j) \leq 2 + 2c_B$ . If  $\tau = 0$  there exists stable equilibria*

*such that there are no two disjoint paths between some nodes  $i, j$ .*

*Proof:* First, we are going to show that  $d'(i, j)$  is finite (step 1). Then, in the second step, we shall bound  $d'(i, j)$  from above.

Note, that if there aren't two disjoint paths between players  $i$  and  $j$ , it is beneficial for both players to add the link  $(i, j)$  and form a path.

Step 1: If  $i$  and  $j$  are disconnected, then it is worthy for both of them to establish the link  $(i, j)$ . Therefore, there exists a path from  $i$  to  $j$  and  $d(i, j)$  is finite. Assume  $d(i, j) \neq 1$ . If there aren't two disjoint paths between nodes  $i$  and  $j$ , it is worthy for both nodes to establish a direct link. Hence if  $d(i, j) \neq 1$  the distance  $d'(i, j)$  is finite. We now discuss the case  $d(i, j) = 1$  and show there must exist an additional disjoint path, so  $d'(i, j)$  is finite. We prove by negation.

Take node  $x \neq i, j$ . As before, both  $d(i, x)$  and  $d(x, j)$  are finite. Consider the following two cases:

A)  $d(i, x) = 1, d(x, j) = 1$ . Then, the trajectory  $(i, x, j)$  is disjoint from the path  $(i, j)$ , and it is therefore worthy for both player  $i$  and player  $j$  to establish the link  $(i, j)$ . Hence  $d'(i, j)$  is finite.

B)  $d(i, x) = 1, d(x, j) \neq 1$  or  $d(i, x) \neq 1, d(x, j) = 1$ . Without loss of generality, we assume the first case. According to the previous discussion, there are at least two disjoint paths from  $x$  to  $j$ . Since they are disjoint, only one of them may contain the edge  $(i, j)$ . Therefore, there exist a path  $(i, x, \dots, j)$  that is disjoint from the edge  $(i, j)$  and therefore  $d'(i, j)$  is finite.

C)  $d(i, x) \neq 1, d(x, j) \neq 1$ . Applying the same reasoning as in B), there exists a path from  $i$  to  $x$  that is disjoint from the edge  $(i, j)$ . Likewise, there exists a path from  $j$  to  $x$  that is disjoint from the edge  $(i, j)$ . Therefore there exist a path from  $i$  to  $j$  that is disjoint from  $(i, j)$ .

Step 2: Step 1 has shown that every two nodes are connected by a cycle. Let us assume the longest shortest cycle connecting two nodes is of length  $l > 2 + 2c_B > 3$ , namely there exists a cycle  $(x_0, x_1, x_2, \dots, x_{\lfloor l/2 \rfloor}, \dots, x_l, x_0)$ .

By establishing the link  $(x_0, x_{\lfloor l/2 \rfloor})$  the cost of player  $x_0$  due to distances from other players is reduced by at least  $(l^2 - 1 + \text{mod}(l+1, 2)) / 4$ , hence the link will be established as

$$\Delta C(x_0, E + (x_0, x_{\lfloor l/2 \rfloor})) \leq (l^2 - 1 + \text{mod}(l+1, 2)) - c_B \leq 0$$

and similarly  $\Delta C(x_0, E + (x_0, x_{\lfloor l/2 \rfloor})) \leq 0$ . Since  $d(i, j) \leq d'(i, j) \leq l$  the proof is complete.  $\blacksquare$

If there exist players with no two disjoint paths in an equilibrium, then the social cost becomes unbounded. This immediately results in an *unbounded* Price of Anarchy, as indicated by the following theorem.

**Theorem 10.** *Consider a network such that  $T_B \gg T_A \gg 1$ . The Price of Anarchy is as follows:*

A) *In a setting with asymmetric reliability requirements ( $\tau = 0$ ): unbounded.*

B) In a setting with symmetric reliability requirements ( $\tau \neq 0$ ): bounded by  $o(c)$ .

*Proof:* A) We shall prove this by showing that there exists a stable equilibrium with unbounded social cost. Consider a network in which the major players (type-A players) form a clique, while every minor player is connected only to a single type-A player  $j \in T_A$ . We shall show that this network is stabilizable. As discussed before, the type-A clique is stable. Consider  $i \in T_B$ . By forming the link  $(i, x)$  the change of cost of player  $x \in T_A$ ,  $x \neq j$  is

$$\Delta C(x, ix) = c_A - 1 > 0.$$

Hence, this link will be formed. No additional links between minor players will be established, since such a link only reduces the distance between the participating parties by one,  $c_B - 1 > 0$  and does not provide an additional disjoint path to the type-A clique. Therefore, the social cost is  $\mathcal{S} = \omega(\mathcal{Q}) \rightarrow \infty$ .

B) The total cost due to the inter-connectivity of the type-A clique is identical for all link stable equilibria and is  $|T_A|(|T_A| - 1)(c + (1 + \delta/2)A)$ . This cost is composed of  $|T_A| - 1$  links per node, and the distance cost to every other major nodes,

$$d(i, j) + d'(i, j) = 1 + \delta/2.$$

Next, we evaluate the cost due to the type-B nodes inter-distances. According to Lemma 9, both  $d(i, j) \leq 4c_B$  and  $d'(i, j) \leq 4c_B$ , so the cost due to the inter-distances between a type-B player and every other player is bounded from above by  $4c_B(|T_A| + |T_B|)$ . When summed up over all minor players, this contributes a term  $4c_B|T_B|(|T_A| + |T_B|)$  to the social cost. Likewise, the cost of links that at least one of their ends is a type B player is at most  $c_B|T_B|(|T_A| + |T_B|)$ .

Therefore, the maximal cost in all link stable equilibria is bounded from above by

$$|T_A|(|T_A| - 1)(c + (1 + \delta/2)A) + 5c_B|T_B|(|T_A| + |T_B|).$$

The optimal network configuration is described in Proposition 8. It is straightforward to evaluate the social cost in this configuration. The distance cost due to inter-distances of the type-B players is

$$2|T_B|(|T_B| - 1),$$

while the cost of all links which connect type-B players to type-A players is

$$|T_B||T_A|(c_B + c_A).$$

Finally, the social cost due to the type-A clique remains the same, so finally we obtain that the minimal social cost is

$$\begin{aligned} \mathcal{S}_{\text{optimal}} &= |T_A|(|T_A| - 1)(c + (1 + \delta/2)A) \\ &\quad + 2|T_B|(|T_B| - 1) + 2|T_B||T_A|(c_B + c_A) \end{aligned}$$

By taking the limits  $|T_A| \rightarrow \infty$ ,  $|T_B|/|T_A| \rightarrow \infty$  we obtain that  $PoA \leq 5c_B/2$ . ■

A stable equilibrium with infinite social cost can be easily achieved by considering a network where all minor players are connected to a single, designated, major player. There exists a single path of at most two hops between every minor player to every major player. However, as the stability requirements are asymmetric, the major players have no incentive to establish additional routes to any minor player, and the reliability requirements of the minor players remain unsatisfied.

#### A. Monetary transfers

The previous discussion assumed that a player cannot compensate other players for an increase in their costs. Yet, contracts between ASs often do involve monetary transactions. Accordingly, in this subsection we shall highlight the additional insights that are obtained when utility transfers are permissible.

Our first result indicates that, in this setting, in contrast to the previous setting, there always exists a fallback route between every two players, regardless of the symmetric or asymmetric nature of the additional survivability constraints. If monetary transfers are feasible, players may compensate other players for the cost of additional links such that all the additional constraints are satisfied. Hence, symmetry is less important than in the previous scenario. Furthermore, this result suggests that every player is connected to every other player by a cycle. The following proposition shows that the maximal cycle length decays with the number of major players. As the number of ASs increases in time, this predicts that this length should decrease in time. We shall verify this prediction in Section V.

**Proposition 11.** Assume  $1 < c < A/2$ . Then, every two players are connected by a cycle, and the maximal cycle length connecting a major player to a minor player is bounded by

$$\max \left\{ 2 \left( \left\lfloor \sqrt{(A|T_A|)^2 + 5c} - A|T_A| \right\rfloor + 1 \right), 4 \right\}.$$

*Proof:* Lemma 9 showed that the maximal distance between players is bounded. We are going to tighten this result in the regime where monetary transfers are feasible. Denote the maximal distance between type-A player and a type\_B player by  $k_A$ .

First, we are going to show that the maximal cycle length connecting a major player and a minor player is  $2k_A + 1$ . This follows from a simple geometric argument. Consider two players,  $i \in T_A$  and  $j \in T_B$ . Assume  $k_A > 2$ . If the cycle length is  $2k_A + 2$  or greater, there exists a type-B node that its distance  $k_A + 1$ , in contradiction to the assumption that the maximal distance between a major player and a minor player is  $k_A$ . Denote maximal distance between two minor players by  $k_B$ . A similar argument shows that maximal cycle length between two minor players is  $2k_B + 1$ .

Next, are going to show that the maximal distance connecting a major player and a minor player is  $k_A \leq l \leq \max \left\{ 2 \left\lfloor \sqrt{(A|T_A|)^2 + 5c} - A|T_A| \right\rfloor, 2 \right\}$ . We prove by negation. Assume that the distance between player  $j \in T_A$

and  $i \in T_B$  is  $l$ . Denote the nodes on the path as  $(x_0 = i, x_1, x_2, \dots, x_l = j)$ . Then, by establishing a link between them, the distance between  $j$  and  $\{x_0, x_1, \dots, x_{\lfloor l/2 \rfloor}\}$  (similarly, and distance between player  $i$  and players  $\{x_{\lceil l/2 \rceil}, \dots, x_{l-1}\}$ ) is reduced. In addition, player  $i$  reduces its distance to every node of the type-A clique by  $l$ . Lemma 25 of [21] shows that the total reduction in distance is  $(l^2 - 1 + \text{mod}(l + 1, 2)) / 4$ . Then, by establishing the link  $(i, j)$  we have

$$\Delta C(i, E + ij) + \Delta C(j, E + ij) \leq c_A + c_B - 2(l^2 - 1) - l|T_A|$$

and as  $l \geq 2 \left\lfloor \sqrt{(A|T_A|)^2 + 5c} - A|T_A| \right\rfloor$  this expression is negative. Therefore, the link will be established, and the maximal shortest distance between a major player and a minor player is smaller than  $l$ . This concludes the proof. ■

Our second result is based on the first one, and shows that the price of anarchy is bounded. In fact, as the network grows, Proposition 11 also indicates that its diameter *shrinks*. Therefore, in the large network limit, the price of anarchy is bounded by a constant.

**Proposition 12.** *The price of anarchy is bounded by  $o(c)$ . Furthermore, if  $|T_B| \gg 1, |T_A| \gg 1$  then the price of anarchy is upper bounded by 2.*

*Proof:* In Proposition 11's proof we showed that the maximal distance between a major player a minor player is bounded by

$$l \leq \max \left\{ 2 \left\lfloor \sqrt{(A|T_A|)^2 + 5c} - A|T_A| \right\rfloor, 2 \right\}$$

It immediately follows that in the large network limit, as  $l \leq 2$ , every player is directly connected to a player that, in turn is connected to the type-A players. The latter can be either a major player, as part of the clique, or a minor player that is connected to every player in the major player's clique. Note that a link between any two players  $y_i$  and  $y_j$  reduces the social cost, since it at least lowers the costs of  $y_i$  of  $y_j$  and can only reduce the costs of other players. Therefore, we can bound the social cost by a configuration in which every type-B player has the minimal number of links, namely two, is at distance two from every major player.

Therefore, the worst social cost is bounded by

$$\begin{aligned} \mathcal{S} \leq & |T_A| (|T_A| - 1) (c + (1 + \delta/2) A) \\ & + 2c|T_B| + 2|T_B| (|T_B| - 1) \end{aligned}$$

Comparing this bound with  $\mathcal{S}_{\text{optimal}}$ , as derived in Proposition 8, in the limit  $|T_A| \rightarrow \infty, |T_B|/|T_A| \rightarrow \infty$  completes the proof. ■

#### IV. DYNAMIC ANALYSIS

The Internet undergoes continuous transformations, such as the emergence of new ASs, or formation of new traffic contracts. In fact, it may very well be out of equilibrium. Therefore, a static analysis of the equilibrium points must

be accompanied by dynamic analysis. Accordingly, our main focus in this section is to identify prevalent network motifs [24], i.e., small sub-graphs that emerge during the natural evolution of the network. In Section V we shall show that these motifs are indeed ubiquitous in the real AS topology, and the frequency of their occurrences is few folds more than expected in a random network.

While there are many possible equilibria, we shall show that convergence occurs only to *just a few*. We shall also show that the convergence time is short, namely *linear* in the number of players.

We start the discussion by setting up the dynamic framework, as first formulated in [21].

##### A. Setup & Definitions

We split the game into *turns*, where at each turn only a single player is allowed to remove or initiate the formation of links. At each point in time, or turn, the players that already joined the game form a subset  $N' \subset T_A \cup T_B$ . We shall implicitly assume that the cost function is calculated with respect to the set  $N'$  of players that are already present in the network. Each turn is divided into *moves*, at each of which a player either forms or removes a single link. A player's turn is over when it has no incentive to perform additional moves. Note that disconnections of several links can be done unilaterally and hence iteratively.

**Definition 13.** Dynamic Rule #1: In player  $i$ 's turn it may choose to move  $m \in \mathcal{N}$  times. In each move, it may remove a link  $(i, j) \in E$  or, if player  $j$  agrees, it may establish the link  $(i, j)$ . Player  $j$  would agree to establish  $(i, j)$  iff  $C(j; E + (i, j)) - C(j; E) < 0$ .

According to this definition, during player's  $i$  turn, all the other players will act in a greedy, rather than strategic, manner. For example, although it may be that player  $j$  prefers that a link  $(i, j')$  would be established for some  $j' \neq j$ , if we adopt Dynamic Rule #1 it will accept the establishment of the less favorable link  $(i, j)$ . In other words, the active player has the advantage of initiation and the other players react to its offers. There are numerous scenarios in which players cannot fully forecast other players' moves and offers, e.g., when information is asymmetric or when only partial information is available [25]. In these settings, it is likely that a greedy strategy will become the modus operandi of many players. This is a prevalent strategy also when the system evolves rapidly and it is difficult to assess the current network state and dynamics.

In a dynamic network formation game, a key question is: Can a player temporarily disconnects itself from the graph, only to reconnect after getting to a better bargaining position? Or must a player stay connected? If the timescale in which the costs are evaluated is comparable to the timescale in which the dynamics occur, then, clearly, a player will not disconnect from the network voluntarily. However, if the latter is much shorter, it may, for a very brief time, disconnect itself from the



graph in order to perform some strategic move. The following rules address the two alternative limits.

**Definition 14.** Dynamic Rule #2a: Let the set of links at the current move  $m$  be denoted as  $E_m$ . A link  $(i, j)$  will be added if  $i$  asks to form this link and  $C(j; E_m + ij) < C(j; E_m)$ . In addition, any link  $(i, j)$  can be removed in move  $m$ .

Dynamic Rule #2b: In addition to Dynamic Rule #2a, player  $i$  would only remove a link  $(i, j)$  if  $C(i; E_m - ij) > C(i; E_m)$  and would establish a link if both  $C(j; E_m + ij) < C(j; E_m)$  and  $C(i; E_m + ij) < C(i; E_m)$

According to Dynamic Rule #2a, a player is allowed to perform a strategic plan in which the first few steps will increase its cost, as long as when the plan is completed its cost will be reduced. On the other hand, if the game follows Dynamic Rule #2b, then a player's cost must be reduced *at each move*, hence such multi-move plan is not possible.

### B. Basic Model - Results

Our first result in this section shows that, during the natural evolution of the network, a “double star” sub-graph, or network motif, often emerges. In the “double star” motif, as depicted in Fig. 2, there exists a primary and a secondary star. All the minor players are connected to the primary star's center. Part of the players are also connected to the other star's center, forming the secondary star. Consider a region where it is immensely difficult to establish a link to a major player, either due to geographical distance, link prices or perhaps additional physical links are simply not accessible. Nevertheless, in order to maintain a reliable connection, there must be at least two links that connect this region to the Internet backbone via some major players. In order to provide a stable, fault tolerant service, every player in this region will form links with the players hosting the endpoints of these links, forming the double star sub-graph. Assume now that link prices reduce over time, or that the importance of a fast connection to the Internet core increases in time. In this case, players may decide to establish direct links with the major players, and remove either one of both links connecting them to the star centers. Note though, that players will be reluctant to disconnect from the star center if the number of nodes in the star is large.

Moreover, the next theorem also shows that, eventually, and fairly quickly, the system will converge to either the optimal stable state, or to a state in which the social cost is a low multiple of the optimal social cost.

**Theorem 15.** Assume symmetric reliability requirements, i.e.,  $\tau = 1$ . If the players follow Dynamic Rules #1 and #2a, then, in any playing order:

A) The system converges to either the optimal stable state, depicted in Fig. 3, or to the network depicted in Fig. 2.

B) In the large network limit, namely, when  $|T_B| \gg |T_A| \gg 1$ , the social costs ratio satisfy  $S/S_{\text{optimal}} < 3/2 + \epsilon$ , with  $\epsilon \rightarrow 0$ .

C) If players play in a uniformly random order, the probability that the system has not converged by turn  $t$  decays

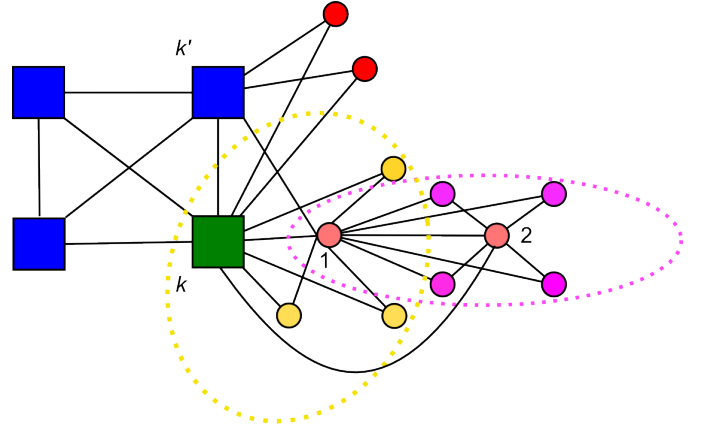


Figure 2. A network configuration which includes a “double-star” structure of minor players. Every node in the primary star (encircled in yellow) is linked to a major player, node  $k$  (in green). A direct link connects the two star centers, denoted by 1 and 2 (in pink). The members in the secondary star (in purple) are connected to both star centers. In addition, there secondary star center is also connected to the major player  $k$ . There might be additional minor players outside the stars (in red). Needless to say, the type-A clique (square boxes) is also present.

$d(i, j)$	$i \in n_1$	$i \in n_2$	$i \in n_3$
$j = 1$	1	1	2
$j \in S_1$	2	2	2
$j = 2$	2	1	2
$j \in S_2$	2	2	3
$j \in L$	2	2	2
$j = k$	1	2	1
$j = k'$	2	2	1
$j \in D_2, j \neq k, k'$	2	2	2
$j \in T_A, j \notin D_2$	2	3	2

Table I  
THE SHORTEST DISTANCE  $d(i, j)$  BETWEEN TWO NODES, AS DISCUSSED IN PROP. 15. NOTE THAT IF  $k' \in D_2$ , THAN COLUMN 9 APPLIES INSTEAD OF COLUMN 8 FOR  $i \in S_2$ .

exponentially with  $t$ . Otherwise, if every player plays at least once in  $O(N)$  turns, convergence occurs after  $O(N)$  steps.

*Proof:* We claim that the network, at any time, has the following structure (Fig. 2): A type-A players' clique, and at most two type-B star centers  $1, 2 \in T_B$ . The larger ball's center is labelled as 1, and the smaller is 2. We denote the set of type-B players which are members of the star centered about node 1 (2) as  $S_1$  and (correspondingly,  $S_2$ ). We have  $|S_1| \geq |S_2|$ . In addition, some type-B players might be linked to two nodes  $k, k' \in T_A$ , the set of these type-B players is denoted by  $L$ . The set of type-A players that have a direct link with the star center 1 (star center 2) is denoted by  $D_1$  (correspondingly  $D_2$ ). Players  $\{1, 2, k\}$  form a clique. Assuming the second star exists, it is connected to either players 1 or  $k'$  (or both). Finally, there may be additional links between players in  $L$  and  $D_2$ . An example of this network structure is presented in Fig. 2.

We prove by induction. After the first three players played the induction base case is true. We first assume that the link  $(1, 2)$  and consider the case  $(1, 2) \notin E$  later. Denote the active



$d'(i, j)$	$i \in n_1$	$i \in n_2$	$i \in n_3$
$j = 1$	2	2	2
$j \in S_1$	2	3	2
$j = 2$	2	2	3
$j \in S_2$	3	2	3
$j \in L$	3	3	2
$j = k$	2	2	2
$j = k'$	2	3	2
$j \in D_2, j \neq k, k'$	2	2	2
$j \in D_1, j \notin D_2$	2	3	2
$j \notin D_1, j \in T_A$	3	3	2

Table II

THE SECOND SHORTEST DISTANCE  $d'(i, j)$  BETWEEN TWO NODES, AS DISCUSSED IN PROP. 15. NOTE THAT IF  $k' \in D_2$ , THAN COLUMN 9 APPLIES INSTEAD OF COLUMN 8 FOR  $i \in S_2$ .

player by  $r$ . Consider the following cases:

1.  $r \in T_A$ : As  $c < A/2$ ,  $r$  will form (or maintain) links with every type-A player. If additional links to minor players are to be formed, it is clearly better for player  $r$  to establish links first with either player 1 or player 2 than to any player in  $i \in S_1$  or  $i \in S_2$ . We first consider the case where  $r \notin D_1 \cup D_2$  and split into two cases:

1A) If no two disjoint paths exists between player  $r$  to players 1 or 2, namely, if  $|T_A| \geq 2$  and either  $|D_1| < \min\{|T_A|, 2\}$  or  $|D_2| \leq \min\{|T_A|, 2\}$ , then it is beneficial for both  $r$  and the star centers to link such that reliability requirements will be satisfied.

1B) If  $D_2 \neq \emptyset$  then by establishing the edge  $(r, 1)$  we have

$$\Delta C(r, E + 1r) = c - \frac{1 + |S_1|}{1 + \delta}$$

while by establishing  $(r, 2)$  the change of cost is:

$$\Delta C(r, E + 2r) = c - \frac{1 + |S_2|}{1 + \delta}. \quad (1)$$

As  $|S_2| \leq |S_1|$ , player  $r$  will prefer to establish first a link with player 1 rather than with player 2. The link will be formed if  $\Delta C(1, E + 1r) \leq 0$ , which is true if

$$\begin{aligned} |D_1| &\leq 1 \\ \text{or} \\ c_B &< A/(1 + \delta). \end{aligned} \quad (2)$$

If condition 1 holds, then player  $r$  will attempt to form a link with the star center 2, and succeed if condition 2 holds as well. Establishing a link to any other type-B node is clearly an inferior option, as if a player had decided to establish links with either node 1 or node 2, an additional link to one of their leafs will only reduce the distance to it by 1 and hence is not a worthy course of action. Therefore,  $D_2 \subseteq D_1$ .

2.  $r \in T_B$  and  $r \neq 1, 2$ . A player may disconnect itself and then choose its two optimal links. Clearly, among the type-A clique, player  $r$ 's best candidates are players  $k, k'$ , while among the type-B players, the cost reduction is maximal by linking to nodes 1, 2. In addition, a link to node  $k$  is at least as preferred a link to  $k'$ , while a link to node 1 is always

preferred over a link to node 2 as long as  $n_1 \geq n_2$ . Therefore, we only need to consider three possible moves by player  $r$  - linking to  $k, k'$  (the red players in Fig. 2), linking to  $k, 1$  or linking to 1, 2.

Using tables I and II we can compare the cost of connecting to nodes 1, 2 versus the costs of connecting to nodes 1 and  $k$ . A simple calculation shows that if the expression

$$A - 1 + (|D_1| - |D_2|)A + \delta \frac{|S_1| - |S_2| + A + A(-|D_2|)}{1 + \delta}$$

is positive, then linking to players 1 and  $k$  is preferred over linking to player 1 and 2. This expression is positive, as  $|D_1| \geq |D_2|$  and  $|S_2| \leq |S_1|$ .

In a similar fashion, we can compare between the choices of linking to nodes 1 and  $k$  or forming links with nodes  $k$  and  $k'$ . If the expression

$$1 + |S_2| - A + \delta \frac{1 - |L| - A(-|D_1|)}{1 + \delta}$$

is positive, then establishing links to 1 and  $k$  is preferred, otherwise the alternative will be chosen. Two special cases are of interest: If  $|S_2| < A - 1$  and  $\delta = 0$ , then every player will prefer to link to  $k$  and  $k'$ . That is, we'll get the optimal configuration. If  $\delta = 1$ ,  $T_A = D_1$ , i.e. node 1 is a member of the clique,  $|L| = 0$  and  $|S_2| \gg 1$ , then a minor player will connect to players 1 and  $k$ . Note that if there are no type-A players present when  $r$  plays, then it must connect to the star centers 1 and 2.

In addition, after  $r$  formed two links, no player  $x \in T_A \cup \{1, 2\} \cup D_1 \cup L$  will agree to form the link  $(x, r)$ , as the induced change of cost is

$$\Delta C(x, E + xr) \geq c_A - 1 > 0.$$

If  $c_B \leq 2$  and  $\delta \rightarrow 0$  then player  $r \in L$  and player  $x \in D_2$  will establish the link  $(x, r)$ . By symmetry, this also happens when  $r \in D_2$  and  $x \in L$ .

3. If  $r = 1$ , then it must maintain links to all nodes in  $S_2 \cup S_1 \cup \{2\}$  (if there are any) in order to satisfy the reliability criteria. Additional links to type-A players will be formed according to the discussion in case 1, while no other links to type-B player will be formed according to the discussion in case 2.

4. If  $r = 2$ , then it must maintain links to all nodes in  $S_2$  (if there are any) in order to satisfy the reliability criteria. A similar calculation to the one in case 2 shows that player 2's best course of action is to remain connected to player

In order to complete the induction proof all that is left is to address the case  $(1, 2) \notin E$ . In this case, player 2 must be connected to at least one additional player  $i \notin S_2$  in order to have two disjoint path to every player in the network. Clearly, its optimal choice is player  $k'$ . In order to maintain reliable path to every  $i \in S_2 \cup \{2\}$  player  $k'$  must agree to form this link. Player 2 will disconnect  $(1, 2)$  if either  $\Delta C(2, E - 12) \leq 0$  or  $\Delta C(2, E - 12 + 2k') \leq 0$ . However, if either of this conditions hold, Tables I and II shows that for every  $i \in T_B/\{2\}$ , it would prefer to link to player  $k'$  rather than

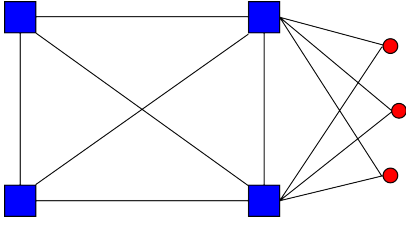


Figure 3. The optimal stable network, as described in Theorem 15.

player 2. In particular, this is also true for every node  $i \in S_2$ . Therefore, as soon as every member in  $S_2$  played once, the star will be empty and no additional star will raise again. Therefore, there will be at most one star left in the network.

This completes the proof on the momentarily structure of the network. Note that if  $|S_1| = |S_2| = \emptyset$  we obtain the optimal stable solution.

B) Since every minor player has either two or three links, the contribution to the social cost due to minor players' links is  $o(|T_B|)$ , while the contribution to the social costs due to the inter-distances between minor players is  $o(T_B^2)$ . In the limit  $|T_A| \rightarrow \infty, |T_B|/|T_A| \rightarrow \infty$ , the dominant term in the cost function is the term proportional to  $|T_B|^2$ . Both  $d(i, j)$  and  $d'(i, j)$  for  $i, j \in T_B$  are bounded by 3, and comparing this results with the optimal social cost (Proposition 8) we have

$$\frac{S}{S_{\text{optimal}}} = \frac{3T_B^2 + o(T_B^2)}{2T_B^2 + o(T_B^2)} \leq \frac{3}{2} + \epsilon$$

with  $\epsilon \rightarrow 0$  in the limit  $|T_A| \rightarrow \infty, |T_B|/|T_A| \rightarrow \infty$ .

C) The discussion of case 2 shows that in a large network, it is suboptimal for a minor player to be a member of  $S_2$ . Given the the opportunity, it will prefer to become a member in either  $S_1$  or  $L$ . Therefore, after every player has played at least twice,  $S_2 = \emptyset$ , and after every player has played four times the system will reach equilibrium. Lemma 13 in [21] then shows that the probability that the system has not converged by turn  $t$  decays exponentially with  $t$ . ■

In section III we emphasized the importance of symmetry in the reliability requirements for reducing the Price of Anarchy. The next theorem affirms this assertion, and shows that if the constraints are asymmetric, the system converges to a state with an unbounded social cost on a large set of possible dynamics and initial conditions.

**Theorem 16.** *Assume asymmetric reliability requirements, namely  $\tau = 0$ . If the players follow Dynamic Rules #1 and either Dynamic Rule #2a or #2b, then the system converges to a state with an unbounded social cost.*

*Proof:* The idea behind the proof is to show that the reliability requirements of at least one minor player remain unsatisfied, the therefore the social costs, as a sum over individual costs, is unbounded.

First assume that  $c_A > 2$ . We shall now show that at any given turn, the network is composed of a type-A (possibly empty) clique, a set of type-B players  $S$  linked to player  $x$ , acting as a star center, and an additional (possibly empty) set

of type-B players  $L$  connected to the type-A player  $k$ . Player  $x$  is also connected to player  $k$ . Some major player may establish links with the star center  $x$ . The set of these major players is denoted by  $D$ . Note that the cost of every player in either  $S$  or  $L$  is  $\omega(Q) \rightarrow \infty$ , since every path from each of these players to the type-A clique crosses player  $k$ .

We prove by induction. At turn  $t \leq 2$ , this is certainly true. Denote the active player at time  $t$  as  $r$ . Consider the following cases:

1.  $r \in T_A$ : Since  $1 < c_A < A$ , all links to the other type-A nodes will be established or maintained, if  $r$  is already connected to the network. Clearly, the optimal link in  $r$ 's concern is the link with the star center  $x$ . Therefore, player  $r$  will attempt to establish the link  $(r, x)$  if

$$\Delta C(r, E + rx) = c_A - |S| - 1 \quad (3)$$

is negative. If  $c_B < A/(1 + \delta)$  or that  $|D| \leq 1$  then  $x$  will accept this link. Regardless, if player  $r$  has formed the link it will not establish a link with  $i \in S$ , as it only reduces its distance to player  $i$  by a single hop, and

$$\Delta C(r, E + ir) = c_A - 1 \leq 0. \quad (4)$$

If Eq. 3 is positive, then, player  $r$  has no incentive to establish a link with any  $i \in S$ , as  $|S| \geq 1$  and the by establishing  $(i, r)$  the sole change is the reduction of  $d(i, r)$  from 3 to 1,

$$\Delta C(r, E + ir) = c_A - 2 \geq c_A - |S| - 1 = \Delta C(r, E + rx) \geq 0.$$

Eq. 4 also shows that if  $r \neq k$ , it will not form a link with  $i \in L$ , while if  $r = k$  it may not remove the link  $(k, i)$  as otherwise  $i$  is disconnected.

2.  $r \in T_B, r \neq x$ : First, assume that  $r$  is a newly arrived player, hence it is disconnected. Obviously, in its concern, a link to the star's center, player  $x$ , is preferred over a link to any other type-B player. Similarly, a link to a player  $k$  is preferred over a link to any other type-A player (or if  $L = \emptyset$  and  $D = T_A$ , equivalent to a link to any other type-A player). Therefore,  $r$  first link choice would be either  $(r, k)$  or  $(r, x)$ . In other words,  $r \in L$  or  $r \in S$ . If  $r \in L$ , than no  $i \in T_A$  will agree to establish a link with  $r$ , as it only reduces its distance from  $r$  by one hop, and doesn't alter  $d(i, x)$  for any other  $x$ . Similarly, if  $r \in S$  than

$$\Delta C(i, E + ri) = c_A - 2 > 0$$

and the link would not be formed. Likewise, no link between  $i \in L$  and  $j \in S$  may be formed, as

$$\Delta C(i, E + ij) = c_B - 2 > c_A - 2 > 0$$

3.  $r = x$ , the star's center:  $r$  may not remove any edge connected to a type-B player and render the graph disconnected. On the other hand, the previous discussion shows it will not establish additional links to nodes in  $L$ .

Note that the reliability requirements of players in  $L \cup S$  are invalidated. Therefore, the cost of every player  $j \in L \cup S$  is at least  $Q$ , and we have  $Q \rightarrow \infty$ . Hence, at any given

turn, as soon as either  $|L| > 1$  and  $|S| \geq 1$ , the social cost is unbounded.

If  $c_A < 2$  a link between player  $i \in T_A$  and player  $r \in T_B$  will be formed. However, as soon as player  $i$  becomes the active player, it is beneficial for it to remove the link  $(i, r)$  and establish a link to the star center  $(i, x)$  instead. Therefore, after player  $i$ 's turn, the cost of player  $r$  is again  $\omega(Q)$ . In a similar fashion, if  $c_B < 2$  than player  $i \in S$  and player  $j \in L$  may establish the link  $(i, j)$ . This does not affect any other player and while it may reduce their costs, it does not provide them with two disjoint paths to the type-A clique, as they both must traverse player  $k$  in order to access it. Therefore, their costs is still  $\omega(Q)$ . ■

Although, at first sight, it might seem that this result is due to the greedy and myopic choice of edges, it is possible to show that there are cases in which this result holds even when players may pick their links strategically rather than in a greedy manner. For example, if  $c_A > 2$  and the number of minor players is odd, than a similar analysis shows that a player in  $S \cup \{x\}$  will establish a link to a single player in  $L$ . However, as the number of minor players is odd, there will always be an unmatched player in either  $L$  or  $S$  and its cost will be  $\omega(Q)$ .

### C. Monetary Transfers

Recall that under the presence of monetary transfers, players  $i$  and  $j$  will agree to establish an edge if  $\Delta C(j, ij) + \Delta C(i, ij) < 0$ . Nevertheless, it may be that during the active player turn there are a few links that satisfy this condition, and the player must prioritize them. Each player's decision is myopic, and is based solely on the current state of the network. Hence, the order of establishing links is potentially important. Needless to say, a player's preference order will depend on the link prices. While there are several alternatives, we adopt the following preference order [21]:

Denote the active player as player  $i$ . Each link  $(i, j)$  carries different utility in player  $i$ 's respect. It is reasonable to assume that a link with a lower "connection value" will be priced lower, so that the link with the least connection utility will be marked with the lowest price. In fact, one may assume its price will be as much as the implied cost of the other party of this link. We will denote this price as  $P^*$ . Every other player  $x$  will use this value and demand an additional payment from player  $i$  for the link  $(i, x)$ , as it is more beneficial for player  $i$ . Formally:

**Definition 17.** "Strategic" Pricing mechanism: Set  $j^*$  as the node that maximizes  $\Delta C(i, E + ij^*)$ . Set  $P^* = \max\{-\Delta C(j^*, E, ij^*), 0\}$ . Finally, set  $\alpha_{ij} = \Delta C(i, E + ij) - (\Delta C(i, E + ij^*) + P^*)$ . The price that player  $j$  requires in order to establish  $(i, j)$  is  $P_{ij} = \max\{0, \alpha_{ij}, -\Delta C(j, E + ij)\}$ .

Under this pricing mechanism, there could be many links that carry the same utility. Some of these links have a better connection value, but they come at a higher price. Since all the links carry the same utility, we need to decide on some preference mechanism for player  $i$ . The simplest one

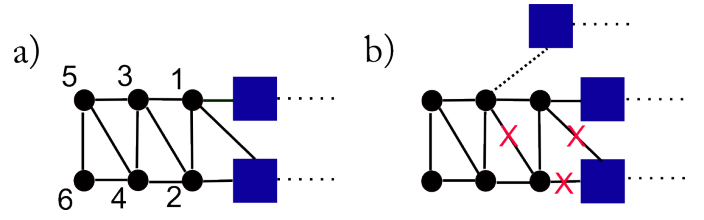


Figure 4. The "entangled cycles" motif. a) Six minor player are connected in an "entangled cycles" subgraph. The first two nodes have direct connection to some major player, and access the rest of the network by the major player's additional links, represented by the dotted line. b) If at some point, a link between a player in this subgraph and some external player is formed (in this example, a major player), some links may be removed without violating the reliability requirement and without increasing the distance cost appreciably. The removed links marked by a red X.

is the "cheap" choice, in which, if there are a few equivalent links, the player will choose the cheapest one. This can be a reasonable choice, as new players cannot spend too much resources, and therefore they will choose the "cheapest" option that belongs to the set of links with maximal utility.

**Definition 18.** Preference order: Player  $i$  will establish links with player  $j$  if player  $j$  minimizes  $\Delta \tilde{C}(i, ij) = \Delta C(i, ij) + P_{ij}$  and  $\Delta \tilde{C}(i, ij) < 0$ . If there are several players that minimize  $\Delta \tilde{C}(i, ij)$ , then player  $i$  will establish a link with a player that minimizes  $P_{ij}$ . If there are several players that satisfy the previous condition, then one out of them is chosen randomly.

We are now at a position to identify an additional network motif, namely, the "entangled cycles". This network motif is composed of a line (i.e., interconnected sequence) of minor players' nodes, with some cross-links between the nodes along this line, breaking the hierarchy (Fig. 4). The "entangled cycle" of length three is the "feedback loop" motif, which was previously found to exist in a higher frequency than expected in the Internet graph [24].

When a new minor player arrives, it will choose the two cheapest links and will connect to the corresponding players. Clearly, its costs due to the distance from the rest of the network will be the highest. As such, when the next player arrives, it will offer the lowest link price. The new arrival will link to it and to one of its provider. The process will repeat, until, at some point, an existing player will decide that this growing branch is too far from it, and will connect to one of the nodes along this "entangled cycles". At this point, this subgraph will be over-saturated with links as players may utilize this link to access the Internet core. Hence, some links will be removed (see, for example Fig. 4(b)). The set of the links that will be removed depends heavily on the playing order and the temporary network structure. Nevertheless, some cross-links may remain in order to satisfy reliability constraints. This explains the following result.

**Theorem 19.** Assume the number of major players is at least 10. Denote the distance cost of player  $i \in T_\beta$ ,  $\beta \in \{A, B\}$ ,

as  $D(i)$ , namely

$$D(i) = C_\beta(i) - c_\beta \deg(i) \cdot x_i$$

Assume that a subset  $W$  of minor player first join the game and play consecutively and that the two players with the maximal distance cost are adjacent. Then:

A) These players will form an “entangled cycles” structure of length  $l$ , as depicted in Fig. 4, and

$$l \leq 2\sqrt{(A|T_A|)^2 + 5A - 2A|T_A|}.$$

B) The “entangled cycles” structure is semi-stable, in the following sense: If, at some later turn, there exist players  $j \notin W$ ,  $i \in W$  such that the link  $(i, j)$  is formed (Fig. 4(b)), then some links in the “entangled cycles” structure may be removed in subsequent turns.

*Proof:* A) Assume that at time  $t$  a subset of  $W = \{x_1, x_2, \dots, x_m\}$  minor players first join the game. Denote the set of players that are currently connected to the network by  $N'$ . We denote by  $n_A$  ( $n_B$ ) the number of type A players (correspondingly, type-B player) at that moment. Let us denote the players with the highest distance cost term as  $x_0$  and  $x_1$ , namely,

$$\begin{aligned} x_0 &= \arg \max_{i \in N'} D(i) \\ x_{-1} &= \arg \max_{i \in N' \setminus \{x_{-1}\}} D(i) \end{aligned}$$

where for simplicity we assumed that  $x_{-1}, x_{-2}$  are minor player (type-B players). According to the “strategic” pricing mechanism a new player will establish links with these players, as they will offer the cheapest links. Note that by connecting to these players, its survivability requirements are satisfied, as each of these players maintain two disjoint path to every major player (if the reliability requirements are asymmetric) or to all other players (in case of symmetric reliability requirements).

We are now going to prove by induction that player  $x_j$  will first connect to players  $x_{j-1}$  and  $x_{j-2}$ . We shall prove by this by proving that the distance costs of  $x_{j-1}$  and  $x_{j-2}$  are maximal.

First, note that for every player  $x_{j'} = 1 \dots j-1$ , we have  $D(x_{j'}) \geq D(x_{j'-1})$  and  $D(x_{j'}) > D(x_{j'-2})$ , since the path from  $j'$  to every player in  $N'$  pass through  $x_{j'-1}$  or  $x_{j'-2}$ , and  $N' \gg l > |W|$ . Therefore, in order to show that players  $x_{j-1}$  and  $x_{j-2}$  have the highest distance cost, it is sufficient to show that  $D(x_{j-1}) > D(y)$  for every  $y \in N'$ . For every player  $i \in N'$  we have

$$\begin{aligned} d(x_{j-1}, i) &\geq d(x_0, i) + \lfloor j/2 \rfloor \\ d'(x_{j-1}, i) &\geq d'(x_0, i) + \lfloor j/2 \rfloor \end{aligned}$$

since the path that connects player  $x_{j-1}$  to the any player  $i \in N'$  crosses either player  $x_0$  or  $x_{-1}$ , which are adjacent. Therefore, the distance cost of player  $x_{j-1}$  due to its distance from every player  $i \in N'$ , denoted by  $\tilde{D}(x_{j-1})$  is at least grater than the corresponding distance cost of player  $x_0$ ,  $\tilde{D}(x_0)$  by at least  $A \lfloor j/2 \rfloor n_A + \lfloor j/2 \rfloor n_B$ . Note

that  $\tilde{D}(x_0) \geq \tilde{D}(y)$  for every  $y \in N'$  according to the definition of  $x_0$ . However, player  $x_{j'-1}$  may be closer to players  $\{x_1, x_2, \dots, x_{j-2}\}$  than player  $y$ . Denote the maximal distance between any two players by  $r$ . We have,

$$D(x_{j-1}) - D(y) \geq A \lfloor j/2 \rfloor n_A + \lfloor j/2 \rfloor n_B - r \cdot j$$

where  $r$  is the maximal distance between two players. In proposition 11 it was shown that

$$\begin{aligned} r &\leq 2 \left\lfloor \sqrt{(A|T_A|)^2 + 5c} - A|T_A| \right\rfloor \\ &\leq 2\sqrt{(A|T_A|)^2 + 5A - 2A|T_A|} \end{aligned}$$

Hence,

$$D(x_{j-1}) - D(y) \geq A \lfloor j/2 \rfloor n_A + \lfloor j/2 \rfloor n_B - r \cdot j$$

As  $|T_A| > 10$  this expression is positive. Hence, links will be established as stated. This shows that at the time player  $x_j$  joins the game, the distance cost of player  $x_{j-1}$  is maximal. A similar calculation shows that at this turn, the distance cost  $D(x_{j-2})$  is maximal in the set  $\{D(i) | i \in N' \cup \{x_1, \dots, x_{j-2}\}\}$ .

We have thus far shown that the first two links of a new player  $x_j$  are to players  $x_{j-1}$  and  $x_{j-2}$ . Nevertheless, as the “entangled cycles” motif grows, the incentive of other players to connect to players in it increases. At some point, player  $j$  may be able to form links with players not in  $W$ , or an active player  $r \notin W$  may decide to connect to some player in  $W$  (or vice versa for  $r \in W$ ). In either of these cases, players in the “entangled cycles” motif may have three disjoint paths to other players, and may therefore remove a link, should they deemed to do so. In this case, the “entangled cycles” motif will be diluted. ■

This theorem shows that reliability is a major factor in breaking up tree hierarchy in the Internet. In addition, it also hints that the hierarchical structure does not break frequently in the top levels of the Internet, but rather mostly in the intermediate and lower tiers. Note that according to Theorem 19(B), in large networks the length of the “entangled motifs” is short. Therefore, we do not expect to see excessively long structures, but rather small ones, having just a few ASs.

## V. DATA ANALYSIS

In this section we compare our theoretical predictions with the real-world inter-AS topology graph [26]. We classified ASs to major players and minor players according the popular CAIDA ranking [27]. For the sake of comparison with previous work [21], we classified the major players as the top 100 ASs according to this ranking.

In Section III-A, we showed that, by allowing monetary transfers, the maximal cycle length connecting a major player to a minor player depends inversely on the number of major players. As the number of ASs increases in time, it is reasonable that the number of major player grows as well. Hence, we expect that the length of the shortest cycle connecting a major player to a minor player will decrease in time. Fig. 5 shows the mean cycle length connecting one of the secondary

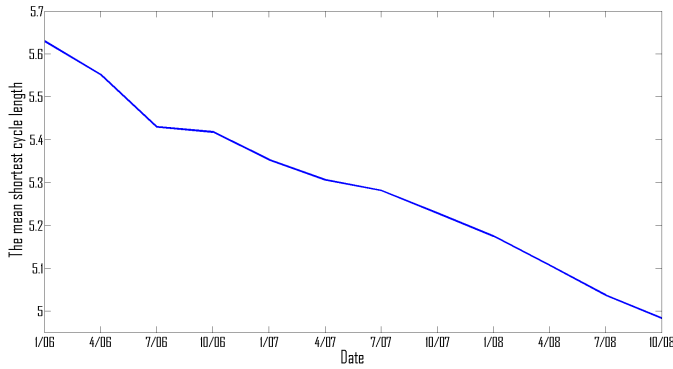


Figure 5. The mean length of the shortest cycle connecting a major player and a minor player as function of time, from January 2006 to October 2008. The length decrease as time passes and the network grows, in agreement with our model.

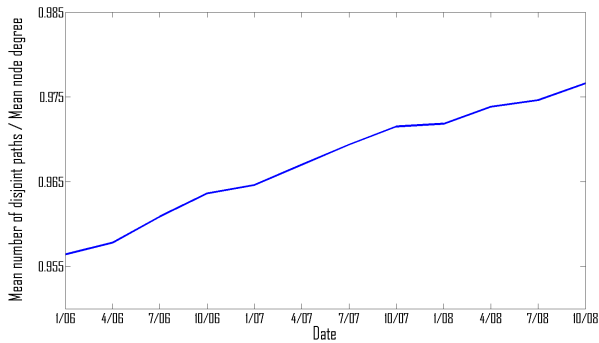


Figure 6. The ratio of the mean number of disjoint paths connecting a minor player to the core to the mean degree of a minor player. This ratio is above 0.95, and increases in time, showing that additional links are likely to part of disjoint paths to the core. This ratio is presented as a function of time from January 2006 to October 2008.

leading 2000 ASs, ranked 101-2100 in CAIDA ranking, and one of the top 100 nodes. The steady decline of the cycle's length in time is predicted by our model.

Our analysis showed that in most of the generated topologies, the minor players are organized in small subgraphs that have direct connection to the Internet core, namely the major players clique, or the tier-1 subgraph, in agreement with [21]. In order to maintain a reliable connection, in each subgraph there must be at least two links that connect minor players to the core. Indeed, we have found out that the ratio between the mean number of disjoint paths from a minor player to the core and the mean degree of minor players is more than 0.95, and it increases in time (Fig. 6). That is, almost every outgoing link of a minor player is used to provide it with an additional, disjoint path to the core. In other words, a player is more likely to establish an additional link, hence increase its degree, if it supplies it with a new path to the core that does not intersect its current paths.

In section IV we predicted the ubiquity of two network motifs, the “double star” motif (Fig. 2) and the “entangled cycles” motif (Fig. 4). We define the occurrence of a “double star” motif as the existence of a connected pair of nodes,

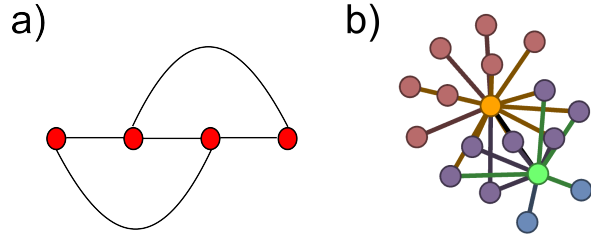


Figure 7. Example of network motifs. a) An “entangled cycle” motif of size four. b) A sample of the “double-star” motif, as found in the real-world inter-AS topology. The star centers are in pink and green. The nodes that are common to both stars are in purple.

each with degree greater than  $m$ , designated as the centers, such that at least  $m$  neighbors of one center are also neighbors of the other center. We generated random networks according to the Configuration Model (CM), in which each node is given a number of stubs according to its degree, and stubs are connected uniformly. Then, we evaluated the mean number of occurrences of this motif in a random CM network with the same number of nodes and the same degree distribution as the real inter-AS topology. For  $m = 2$ , we have found  $28.8K$  occurrences in the real-world inter-AS topology, whereas the mean number of occurrences in the random CM network was only  $5.8K \pm 1.3K$  instances (the  $\pm$  indicates standard deviation). Namely, there are more than four times displays of this subgraph in the Internet than in a random network with the same degree distribution. Chebyshev's inequality provides a bound on the  $p$ -value,  $p < 0.003$ . This low value indicates that it is highly unlikely that a random CM network explains the frequent appearance of this network motif. We have tested the prevalence of this motif with other values of  $m$  and the number of occurrences is consistently a few times more than expected in a random CM network. Our analysis suggests that reliability considerations is one of the factors leading to the increased number of incidents.

The unexpected prevalence of the “feedback loop”, which is a special case of the “entangled cycles” motif, was first reported in [24]. The “feedback loop” motif coincides with the “entangled cycles” motif of length three, and in order to further assess our results we tested for the occurrence frequency of the “entangled cycles” motif of length four. We compared the number of occurrences of this motif in the real-world Internet graph to the expected number of occurrences in a random Configuration Model network. While the number of instances of this motif in the Internet graph was  $27.7M$ , the expected number of occurrences in the random network was only  $1.3M \pm 0.8M$ . As before, the abundance of this network motif, an order of magnitude greater than expected ( $p < 0.001$ , a relaxed bound based on Chebyshev's inequality) provides a positive indication to the implications of survivability requirements.

In summary, we have provided both static and dynamic empirical evidence that conform with our predictions, suggesting the importance of reliability considerations on the structure

and dynamics of the inter-AS topology.

## VI. CONCLUSIONS

While many studies have tried to model the Internet structure, the list of works that explicitly address reliability considerations is much shorter. Furthermore, most of the theoretical models of the Internet structure and dynamics assume homogenous agents, while the Internet is inherently heterogeneous, composed of a wide variety of entities with different business models. In this work, we found that constructing a model that includes these two factors, namely reliability and heterogeneity, may provide important insights on the Internet structure and dynamics.

We first rigorously formulated a model of a network formation game in this context. Our model is flexible, and may be used in a wide variety of settings. It allows for many variations and schemes, for example situations in which failures are frequent or rare, or to account for varying centrality of different types of players. The homogeneity of the model also allows us to describe scenarios in which a fallback route is required only to a subset of players. Indeed, a reasonable AS policy is to require a backup routing path only to the Internet backbone, rather than establishing fault-tolerant routing pathways to every particular Autonomous System.

We established the *Price of Reliability*, which measures the excess social cost that is required in order to maintain network survivability in an optimal stable equilibrium. Surprisingly, we showed that it can be smaller than one, that is, that the additional survivability constraints *add* to the social utility. We have also showed that reliability requirements have disparate effects on different parts of the network. While it may support dilution in dense areas, it facilitates edges formation in sparse areas, and in particular it supports the formation of edges connecting minor players and major players.

In our dynamic analysis we have found the repetitive appearance of small sub-graphs, or network motifs, namely the “entangled cycles” motif and the “double star” motif. Indeed, the number of appearance of these motifs in the real Inter-AS topology surpassed the expected number by a few folds, indicating that additional factors support their formation, and as our analysis shows, survivability is one of them. We have also predicted that the length of the minimal cycle connecting a major player to a minor player should decrease in time. This prediction, too, was verified by a dynamic data analysis.

Finally, while our analysis focuses on the inter-AS topology, it may be applied to other networks as well, that are composed of heterogeneous, rational agents that are required to maintain some reliability aspects. Primary examples are trade networks and MVNO operators in the cellular market.

In this work we have shown that a game theoretic analysis of network formation, which encompasses heterogeneous agents and explicitly addresses survivability concerns, holds promising results. Nevertheless, there are many open questions, such as the following. How does the emerging network handle more than a single failure? Which incentive mechanism will promote

increased reliability of the future Internet? What can a comparative analysis of experimental results on different networks tell us about the players’ strategies in each network? These, and many more, indicate that there is yet a lot to uncover in this intersection between network formation, heterogeneity and reliability.

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